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K. Tazoe, H. Tanaka, M. Oka, & G. Yagawa (Eds)

AN ANALYSIS OF HALF ELLIPTICAL SURFACE CRACK PROPAGATION PHENOMENON WITH SMOOTHED PARTICLE HYDRODYNAMICS METHOD

KOKI TAZOE[1], HIROTO TANAKA[2], MASANORI OKA[3] AND GENKI YAGAWA[4]

[1] Research & Development Center, YANMAR CO., LTD.
2481, Umegahara, Maibara, Shiga, 521-8511, Japan
e-mail: koki_tazoe@yanmar.com

[2] Research & Development Center, YANMAR CO., LTD.
2481, Umegahara, Maibara, Shiga, 521-8511, Japan
e-mail: hiroto_tanaka@yanmar.com

[3] Research & Development Center, YANMAR CO., LTD.
2481, Umegahara, Maibara, Shiga, 521-8511, Japan
e-mail: masanori_oka@yanmar.com

[4] Emeritus Professor, The University of Tokyo and Toyo University
1-32-19, Akatsutsumi, Setagaya, Tokyo, 156-0044, Japan
e-mail: yagawag@gmail.com

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Abstract. The smoothed particle hydrodynamics (SPH) method was applied to the problem of fatigue crack propagation. The stress singularity characteristics at the crack tip and the stress intensity factor were compared between the SPH results and the reference values. The result of half elliptical surface crack propagation analysis showed smooth crack propagation history and the shape of the analyzed fracture surface was close to that achieved by test. Accordingly, it is concluded that the SPH is a useful tool to analyze the linear elastic fracture mechanics and the fatigue crack propagation.

1 INTRODUCTION

The estimation of fatigue damage is essential to ensuring the safety of mechanical structures. Fatigue crack propagation is one of the dominant phenomenon of fatigue damage. Accordingly, some meshing numerical analyses for the crack propagation problem, represented by X-FEM [1], have been investigated. However, in general, it is difficult to use meshing numerical analyses to deal with some complex situations, such as the process of penetrating the thickness of a plate, or connecting multiple cracks or defects. On the other hand, mesh-less numerical analyses, like a particle method, can solve such problems easily.

In this study, smoothed particle hydrodynamics (SPH) method [2] was applied to linear elastic fracture mechanics and fatigue crack propagation. In order to investigate the way of handling linear elastic fracture mechanics with SPH, the stress singularity at the crack tip area in a single edge cracked specimen was analyzed and compared with past numerical results [3].

After that, the half elliptical surface crack propagation phenomenon, one of the fundamental fatigue crack propagation problems, was analyzed and compared with the fatigue crack propagation test results.

2 FRACTURE MECHANICS WITH SPH METHOD

2.1 Some equations of linear elastic fracture mechanics

The magnitude of load acting on a crack is defined as stress intensity factor K. For example, as seen in Fig. 1, in the case of a two-dimensional crack exposed to mean stress σ_0, the distribution of mode I direction stress at the crack tip area has the following singularity.

$$\sigma(r) = K / (2\pi r)^{1/2} \tag{1}$$

This equation regards all areas of the specimen as an elastic field. As seen in this equation, K is the coefficient of the stress distribution function at the crack tip. Namely, the K value is the magnitude of local stress field at the crack tip area, where the fracture really occurs. In the case of Fig. 1, K is defined as the function of crack length a and shape correcting coefficient F, as follows [4].

$$K = \sigma_0 F(\pi a)^{1/2} \tag{2}$$

In the case of fatigue load, as seen in Fig. 2, stress intensity factor range ΔK, which is the difference between maximum K and minimum K, rules the fatigue fracture phenomenon.

$$\Delta K = K_{max} - K_{min} = (1 - R)K_{max} \tag{3}$$

where stress ratio R is the proportion of the minimum load to maximum load. Generally, ΔK is related to fatigue crack growth rate da/dN, which is the increment of crack length per one fatigue cycle, as seen in Fig. 3. In the middle range, the relation has the following Paris-Erdogan law [5].

$$\mathrm{d}a / \mathrm{d}N = C\Delta K^m \tag{4}$$

where C and m are material constant.

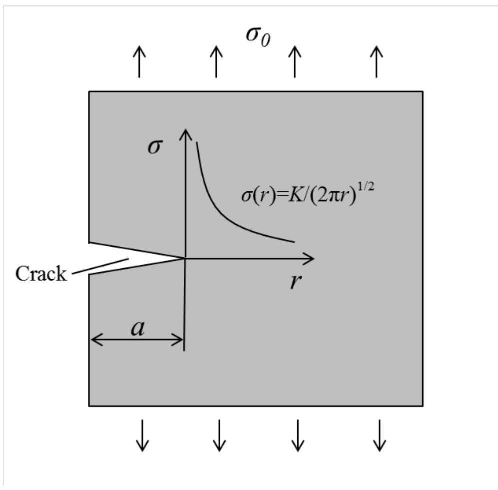


Figure 1 2D mode I crack

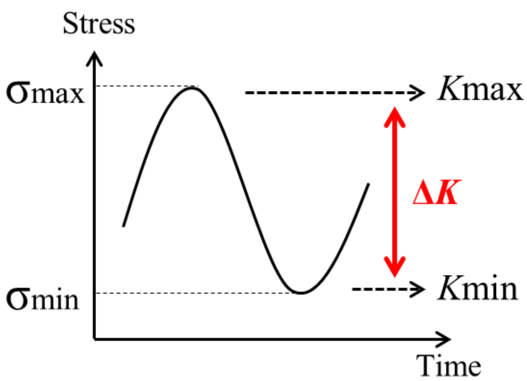


Figure 2 Stress vs. time

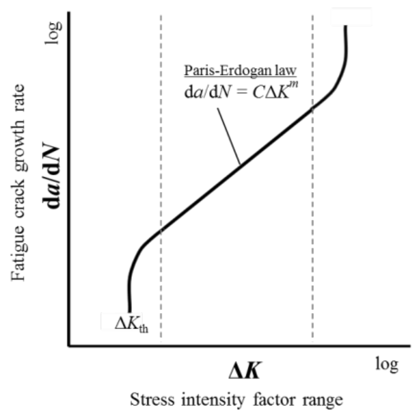


Figure 3 Fatigue crack growth curve

2.2 Stress intensity factor range

In the SPH fatigue crack propagation analysis, K values in all crack tip particles must be calculated. In this study, K values are calculated simply as follows.

$$K = \sigma_1(\pi A)^{1/2} \tag{5}$$

where σ_1 is the principal stress of the crack tip particle and A is the particle size. This equation is derived from substitution σ_1 and 1/2 A for equation (1). This program is used for constant range fatigue load, then, ΔK is calculated by substituting R and equation (5) for equation (3). To verify the validity of this method, stress singularity at the crack tip in a single edge cracked model, which is one of the fundamental crack problems, is analyzed and compared with past numerical analysis.

2.3 Fatigue crack propagation model

Generally, crack propagation analysis with a meshing numerical method can define crack tips in any position. However, SPH analysis cannot be used in the same way because the positions of the particles are restricted by particle size. Therefore in this study, the following damage model is applied to the crack tip particles, and the fracture is defined for every crack tip particle.

$$\Delta D_{i,n} = (\mathrm{d}a / \mathrm{d}N|_{i,n})\Delta N_n / A \tag{6}$$

$$D_{i,n} = D_{i,n-1} + \Delta D_{i,n} \tag{7}$$

where ΔD is the increment of damage, ΔN is the increment of fatigue cycle and D is the total damage value of the particle. D is the dimensionless quantity and the remaining life L for each crack tip particle is defined as follows

$$L_{i,n} = 1 - D_{i,n} \tag{8}$$

ΔN in each analysis step is calculated by equations (5), (4) and (8). After ΔN is obtained, ΔD is calculated by equation (6). The damage in step n $D_{i,n}$ is calculated by the sum of $D_{i,n-1}$ and $\Delta D_{i,n}$. Fracture occurs in the crack tip particle when D is equal to 1, namely L is equal to zero. The fracture particle is removed from the crack tip and a new crack tip is redefined. After that, the next analysis step is calculated.

3 FATIGUE TEST

3.1 Model of cracked specimen

Fig. 4 shows cracked models for SPH analysis. Fig. 4 (a) is a single edge cracked plate model for the stress singularity analysis. Particle sizes are 0.1 mm, 0.05 mm and 0.02 mm, and the numbers of particles are 19,950, 79,900 and 449,750, respectively. Fig. 5 shows a half elliptical surface cracked model for fatigue crack propagation analysis. The particle size is 0.1 mm and the number of particles is 154,810. The cracked areas are modeled by removing particles. Accordingly, the distance between upper crack surface and lower the surface is one particle size. The half elliptical surface cracked model has a triangle shaped initial crack on one of its corners. The support domain size of each analysis is twice the particle size.

3.2 Specimen for fatigue test and material

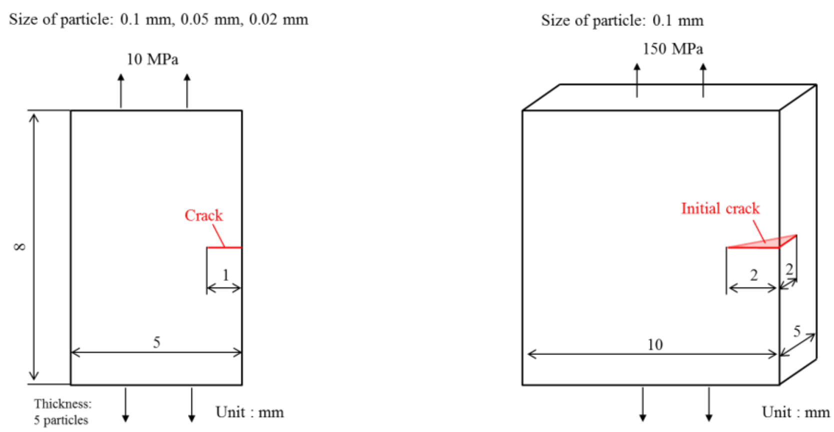
Fig. 5 shows the geometry of the half elliptical surface crack specimen. The specimen has the same initial slit as the model in Fig.4 (b). The initial slit was made by a wire electric discharge machine.

The material used in this study was low carbon steel JIS-S50C. Table 1 and Table 2 show the chemical compositions and mechanical properties of the material respectively. Fig. 6 shows the fatigue crack growth curve of the material. C value is 2.0×10^{-12} and m value is 3.2443.

3.3 Crack propagation test

To compare with the result of the SPH crack propagation analysis, a fatigue crack propagation test with a half elliptical surface crack was carried out. In this study, to visualize the crack shape history, beach marks were introduced to the fracture surface. The marks were introduced by increasing the stress ratio as seen in Fig. 7.

In this test, the fatigue life was not obtained because the stress ratio and load level were changed during the test. The comparison of fatigue life will be carried out with constant loaded fatigue test results in the future.



(a) Single edge cracked plate model (b) Half elliptical surface cracked model

Figure 4 Models for SPH

Table 1: Chemical Compositions of JIS-S50C [mass. %]

C	Si	Mn	P	S	Cu	Ni	Cr	Mo	V
0.48	0.18	0.63	0.014	0.002	0.12	0.07	1.04	0.15	0.01

Table 2: Mechanical propaties of JIS-S50C

Yield stress [MPa]	Tensile stress [MPa]	Elongation [%]	Reduction of area [%]	Vickers hardness*
353	681	33.6	12.6	195

*Force:2 [kgf], Time:30 [sec]

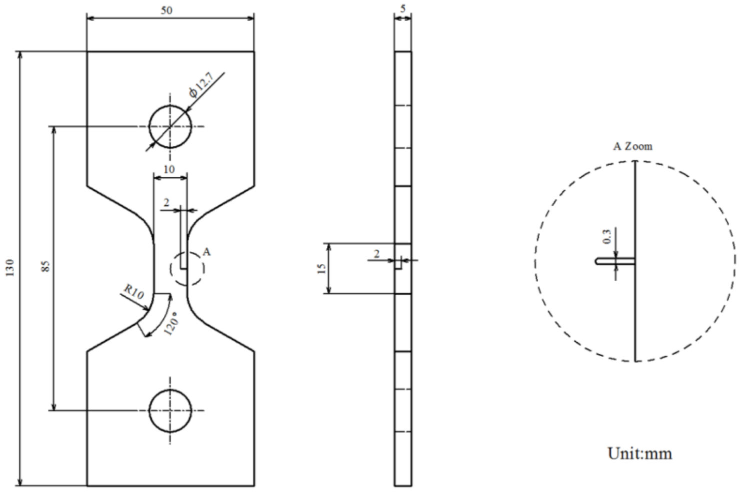


Figure 5 Half elliptical surface cracked specimen

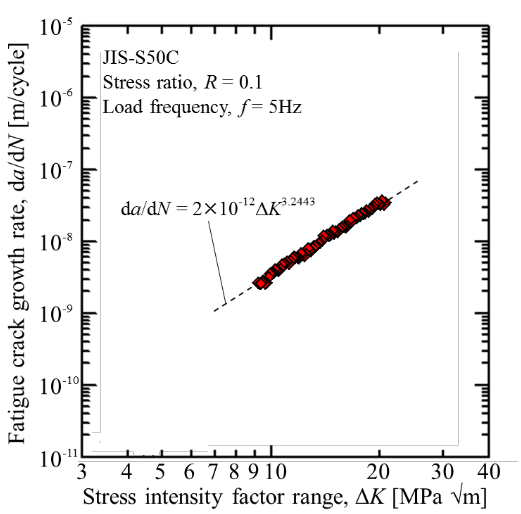


Figure 6 Fatigue crack growth curve of JIS-S50C

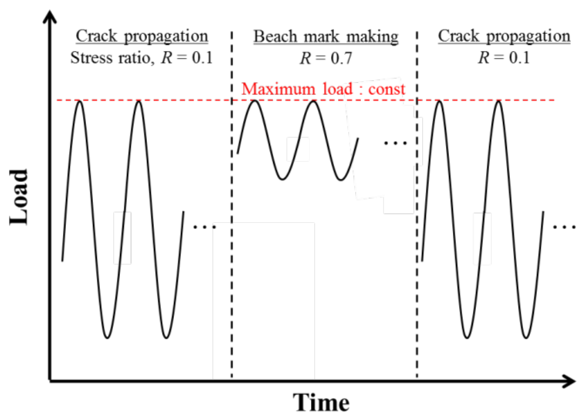


Figure 7 Load vs. time in fatigue crack propagation test

4 ANALYSIS RESULTS

4.1 Stress singularity

Fig. 8 shows the distributions of stress at the crack tip of a single edge cracked model (see Fig. 1), where the SPH results with three different particle sizes and that based on the reference [3] are rather well compared. The K value calculated from the SPH result of particle size 0.02 mm using equation (5) is 0.90 MPa√m, whereas that of the reference [3] is 0.77 MPa√m.

4.2 Fatigue crack propagation

Fig. 9 shows the results of half elliptical surface crack propagation. Fig. 9 (a) shows the crack shape history analyzed by the SPH method. The SPH results show the crack propagation history of a half elliptical surface crack penetrate the plate thickness smoothly. Fig. 9 (b)-(i) shows the original image of the tested fracture surface. In the fracture surface, the beach marks, namely the real crack shape history, are observed clearly. The results of SPH analysis almost trace the tested results, as seen in Fig. 9 (b)-(ii) and Fig. 9 (b)-(iii).

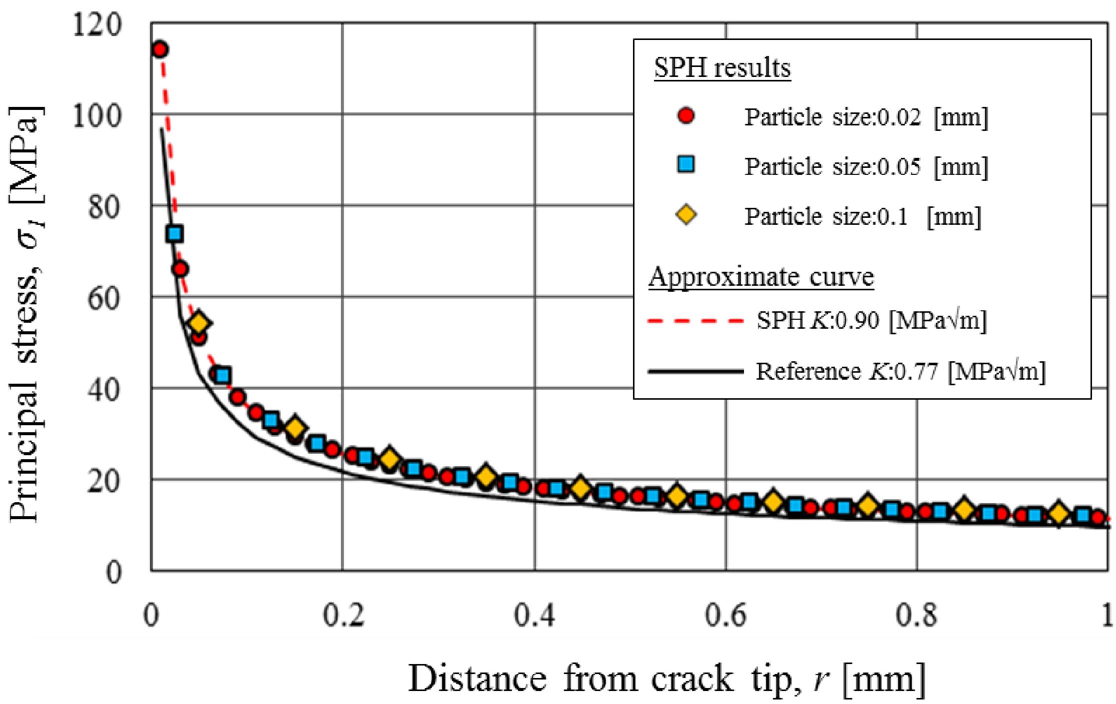
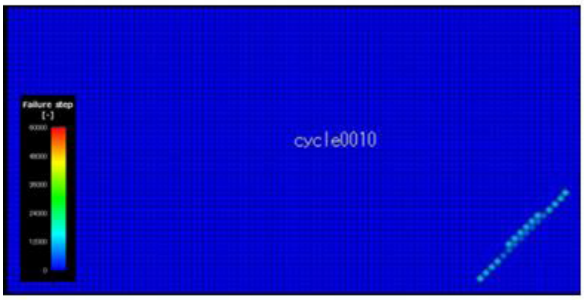
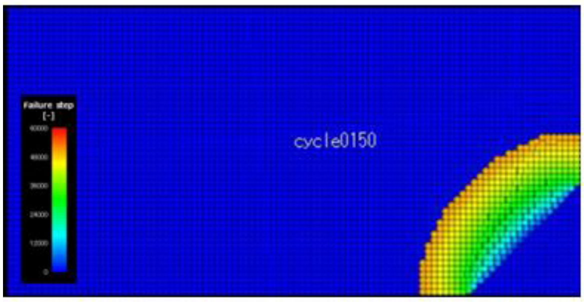


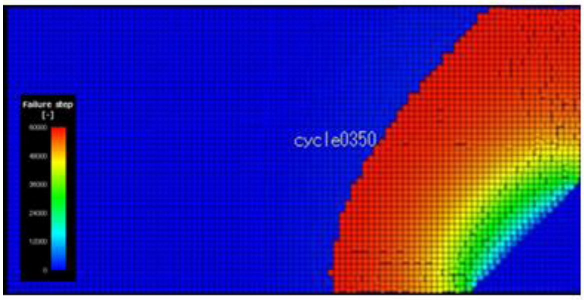
Figure 8 Stress distributions at crack tip



(i) Initial stage

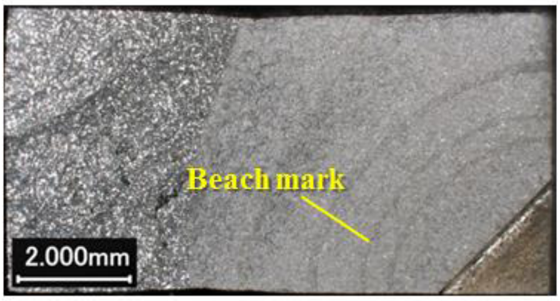


(ii) Middle stage

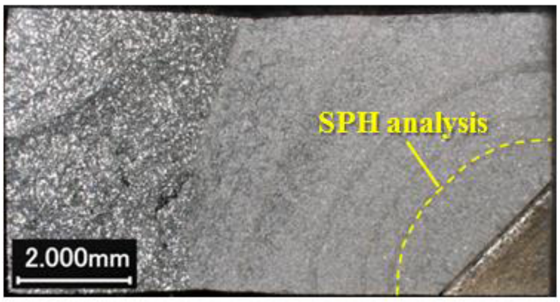


(iii) Final stage

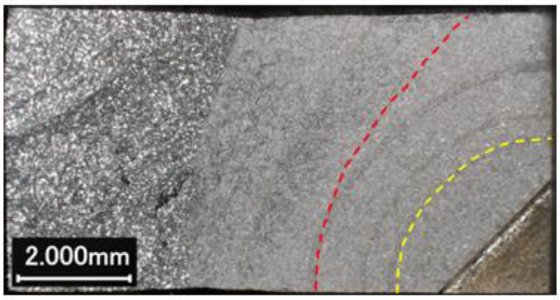
(a) SPH fatigue crack propagation analysis



(i) Original fracture surface



(ii) Fracture surface with SPH middle stage



(iii) Fracture surface with SPH final stage

(b) Fatigue crack propagation test

Figure 9 Fracture surfaces

5 CONCLUTIONS

The following results were obtained.

(a) The SPH is able to calculate stress singularity at the crack tip rather well.

(b) K value by SPH analysis tends to be higher than the reference value.

(c) The half elliptical surface crack propagation is simulated in a smooth manner.

(d) The results of the SPH analysis trace well the beach marks observed at tested fracture specimen.

It is concluded that the SPH method is considered to be useful for analysis of linear elastic fracture mechanics and fatigue crack propagation. The difference of the K values between the SPH result and the reference value as well as the predictions of the fatigue life will be studied in the future.

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